# 1

# *Fundamentals of Magnetic Theory*

This chapter gives a brief review of the basic laws, quantities, and units of magnetic theory. Magnetic circuits are included together with some examples. The analogy between electric and magnetic circuits and quantities is presented. Hysteresis and basic properties of ferromagnetic materials are also discussed. The models of the ideal transformers and inductors are shown.

## 1.1 Basic Laws of Magnetic Theory

The experimental laws of electromagnetic theory are summed up by the Maxwell equations. In 1865, after becoming acquainted with the experimental results of his fellow Englishman Faraday, Maxwell gave the electromagnetic theory a complete mathematical form. We will present specific parts of the Maxwell equations: Ampere's law, Faraday's law, and Gauss's law, which together with Lenz's law are the basis of magnetic circuit analysis. These are the laws that are useful in the design of magnetic components for power electronics.

### 1.1.1 Ampere's Law and Magnetomotive Force

When an electrical conductor carries current, a magnetic field is induced around the conductor, as shown in Fig. 1.1. The induced magnetic field is characterized by its *magnetic field intensity* $H$. The direction of the magnetic field intensity can be found by the so-called *thumb rule*, according to which, if the conductor is held with the right hand and the thumb indicates the current, the fingers indicate the direction of the magnetic field.

The magnetic field intensity $H$ is defined by *Ampere's law*. According to Ampere's law the integral of $\boldsymbol{H}$ [A/m] around a closed path is equal to the total current passing through the interior of the path (note that a line above a quantity denotes that it is a vector):

$$\oint_l \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} \tag{1.1}$$

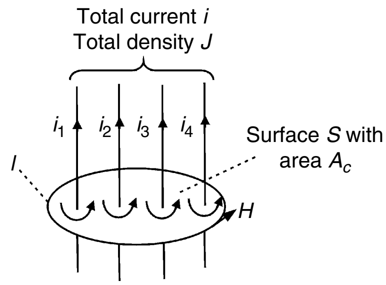


**FIGURE 1.1**
Illustration of Ampere's law. The MMF around a closed loop is equal to the sum of the positive and negative currents passing through the interior of the loop.

where
- $\boldsymbol{H}$ is the field intensity vector [A/m]
- $\mathrm{d}\boldsymbol{l}$ is a vector length element pointing in the direction of the path $l$ [m]
- $\boldsymbol{J}$ is the electrical current density vector [A/m$^2$]
- $\mathrm{d}\boldsymbol{S}$ is a vector area having direction normal to the surface [m$^2$]
- $l$ is the length of the circumference of the contour [m]
- $S$ is the surface of the contour [m$^2$]

If the currents are carried by wires in a coil with $N$ turns, then

$$\oint_l \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} = Ni \tag{1.2}$$

where
- $i$ is the current in the coil
- $N$ is the number of the turns

The terms $\int \overline{H} \cdot \mathrm{d}\bar{l}$ and $Ni$ in Equation (1.2) are equivalent to a source called *magnetomotive force* (MMF), which is usually denoted by the symbol $F$ [A · turns]. Note that the number of turns $N$ does not have dimension, but the value $Ni$ is an actual MMF and not a current. According to Equation (1.1) the net MMF around a closed loop with length $l_c$ is equal to the total current enclosed by the loop. Applying Ampere's law to Fig. 1.1 we obtain

$$\oint_l \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \sum_1^n i = i_1 + i_2 + i_3 + i_4 \tag{1.3}$$

In Fig. 1.1 the reference directions of the current and the $\boldsymbol{H}$ field vector are shown. The magnetic field intensity $\boldsymbol{H}$ leads to a resulting *magnetic flux density* $\boldsymbol{B}$ given by

$$\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H} = \mu \boldsymbol{H} \tag{1.4}$$

where:

$\mu$ is a specific characteristic of the magnetic material termed *permeability*
$\mu_0$ is the *permeability of free space*, a constant equal to $4\pi \times 10^{-7}$ H/m
$\mu_r$ is the *relative permeability* of the magnetic material

The value of $\mu_r$ for air and electrical conductors (e.g., copper, aluminum) is 1. For ferromagnetic materials such as iron, nickel, and cobalt the value of $\mu_r$ is much higher and varies from several hundred to tens of thousands.

The magnetic flux density $B$ is also called *magnetic induction* and, for simplicity, in this book we will use the term *induction* for this magnetic quantity. The vector $\boldsymbol{B}$ is the surface density of the magnetic flux. The scalar value of the total *magnetic flux* $\Phi$ passing through a surface $S$ is given by

$$\Phi = \int_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} \tag{1.5}$$

If the induction $B$ is uniform and perpendicular to the whole surface area $A_c$, then the expression in Equation (1.5) results in

$$\Phi = BA_c \tag{1.6}$$

We have to mention that the expression given by Equation (1.1) is not complete; there is a term missing in the right-hand side. The missing term, which is a current in fact, is called *displacement current* and was added to the expression by Maxwell in 1865. The full form of the law is

$$\oint_l \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} + \frac{\partial}{\partial t}\int_S \varepsilon \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} \tag{1.7}$$

where

$\varepsilon$ is the permittivity of the medium
$\boldsymbol{E}$ is the electric field

Maxwell's correction to Ampere's law is important mainly for high-frequency applications with low current density. In magnetic components for power electronics the expected current density is of the order of at least $J = 10^6$ A/m$^2$. In all normal applications the second term on the right-hand side of Equation (1.7) (the Maxwell's correction) is almost surely not more than 10 A/m$^2$, and can be neglected. Exceptions are the currents in capacitors, currents caused by so-called parasitic capacitances, and currents in transmission lines. This conclusion allows us to use the simplified expression in Equation (1.1) in power electronics magnetic circuit analysis, an approach called the quasi-static approach.

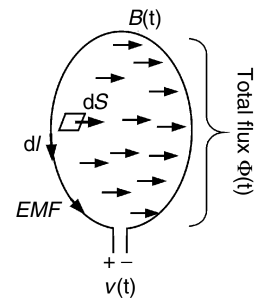


**FIGURE 1.2**
Illustration of Faraday's law. The voltage $v(t)$ induced in a closed loop by a time-changing flux $\Phi(t)$ passing the loop (generator convention).

### 1.1.2 Faraday's Law and EMF

A time-changing flux $\Phi(t)$ passing through a closed loop (a winding) generates voltage in the loop. The relationship between the generated voltage $v(t)$ and the magnetic flux $\Phi(t)$ is given by *Faraday's law*. According to Faraday's law the generated voltage $v(t)$ is

$$v(t) = \frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} \tag{1.8}$$

If we denote the intensity of the electric field as $E$, then Faraday's law is

$$\oint_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} \tag{1.9}$$

Equation (1.9) is valid for the generator convention. For the consumer convention there is no minus sign in it. In this book we use the consumer convention. The positive senses of $\boldsymbol{B}$, $\mathrm{d}\boldsymbol{l}$, $\mathrm{d}\boldsymbol{S}$, and the generated *electromotive force* (*EMF*) are shown by arrows in Fig. 1.2.

Faraday's law is valid in two cases:

- A fixed circuit linked by a time-changing magnetic flux, such as a transformer
- A moving circuit related to a time-stationary magnetic flux in a way that produces a time-changing flux passing through the interior of the circuit.

Rotating electrical machines generate EMF by the latter mechanism.

### 1.1.3 Lenz's Law and Gauss's Law for Magnetic Circuits

*Lenz's law* states that the voltage $v(t)$ generated by a fast time-changing magnetic flux $\Phi(t)$ has the direction to drive a current in the closed loop, which induces a flux that tends to oppose the changes in the applied flux $\Phi(t)$. Figure 1.3 shows an example of Lenz's law.

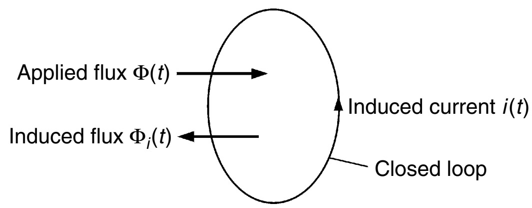


**FIGURE 1.3**
Illustration of Lenz's law in a closed winding. The applied flux $\Phi(t)$ induces current $i(t)$, which generates induced flux $\Phi_i(t)$ that opposes the changes in $\Phi(t)$.

Lenz's law is useful for understanding the eddy current effects in magnetic cores as well in the coil conductors. The eddy currents are one of the major phenomena causing losses in magnetic cores and in coil conductors.

*Gauss's law* for magnetic circuits states that for any closed surface $S$ with arbitrary form the total flux entering the volume defined by $S$ is exactly equal to the total flux coming out of the volume. This means that the total resulting flux through the surface is zero:

$$\oint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0 \tag{1.10}$$

Gauss's law for magnetic circuits is analogous to Kirchoff's current density law for electrical circuits.

## 1.2 Magnetic Materials

Magnetic materials can be classified in three general groups according to their magnetic properties:

- Diamagnetic materials
- Paramagnetic materials
- Ferromagnetic materials

The relative permeability $\mu_r$ of diamagnetic and paramagnetic materials is close to unity. The values of $B$ and $H$ are linearly related for both materials. *Diamagnetic materials* have a value of $\mu_r$ less than unity, which means that they tend to slightly exclude magnetic field, that is, a magnetic field intensity is generally smaller in a diamagnetic material than it would be in a paramagnetic material under the same conditions. The atoms of diamagnetic materials

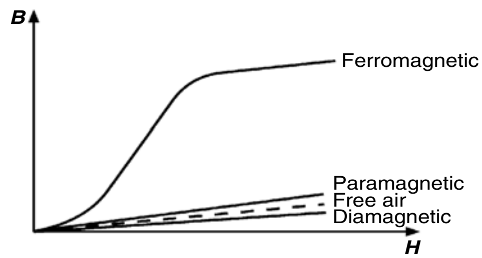


**FIGURE 1.4**
Magnetization curves for different types of magnetic materials. The scale of the magnetization curve of ferromagnetic materials is much higher.

do not have permanent magnetic moments. *Superconductors* are a specific class of diamagnetic materials. In these materials there are macrocurrents circulating in the structure. These currents oppose the applied field and as a result the material excludes all exterior fields. *Paramagnetic materials* have a value of $\mu_r$ greater than unity, and they are slightly magnetized by an applied magnetic field. *Ferromagnetic materials* are characterized by values of $\mu_r$ much higher than unity (10–100,000) [1]. For the design of magnetic components for power electronics, the third type of materials, the ferromagnetic materials, are of real importance, especially ferromagnetic ceramics and metals. Comparison of *B-H* relation of different types of magnetic materials is shown in Fig. 1.4.

### 1.2.1 Ferromagnetic Materials

To understand ferromagnetic materials we will start with the magnetic moments of atoms and the structure of metals. Each electron possesses an electrical charge and its own magnetic (spin) moment. Besides the spin, each electron of the atom has another magnetic moment, a so-called orbital moment, caused by its rotation around the nucleus. In the atoms of many elements the electrons are arranged in such a way that the net atomic moment is almost zero. Nevertheless, the atoms of more than one-third of the known elements possess a magnetic moment. Thus, every single atom of these elements has a definite magnetic moment as a result of the contributions of all of its electrons. This magnetic moment can be associated with an atomic magnet.

In metals there is an interaction between the atoms, which defines the magnetic properties of the total structure. In most cases the atomic moments in the crystal are inter-coupled by coupling forces. If the atomic moments are arranged in parallel with crystal lattice sites, then the moments of the individual atoms are summed up resulting in the *ferromagnetic effect*. The coupling forces in the ferromagnetic materials of technical interest are strong and at room temperature almost all atomic magnets are parallel-aligned. The alignment of the atomic magnets does not occur in the entire structure, but only within certain regions. These regions of alignment of the atomic magnets are called *ferromagnetic domains* or *Weiss domains*. In polycrystalline

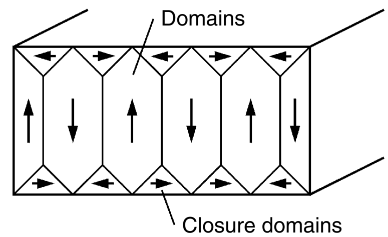


**FIGURE 1.5**
Orientation of domain magnetic moments in the structure of unmagnetized iron.

materials they usually have a laminar pattern. The size of the domains varies considerably, from 0.001 $mm^3$ to 1 $mm^3$. Each domain contains many atoms and is characterized by an overall magnetic moment, as a result of the summing of the atomic magnets. The directions of the domain magnetic moments in an unmagnetized crystal are not completely random among all available directions. The domain magnetic moments are oriented so as to minimize the total external field, and in that way to keep the energy content as low as possible. To follow this rule, adjacent domains have opposite magnetic moments, as shown in Fig. 1.5. The net external field is reduced additionally by so-called *closure domains*, shown in Fig. 1.5.

In every crystal the domains are divided from each other by boundaries, so-called *domain walls* or *Bloch walls*. Across the domain walls the atomic magnetic moments reverse their direction, as shown in Fig. 1.6

The described mechanism of summing the atomic magnetic moments, resulting in spontaneous magnetization of the domains in ferromagnetic materials, is valid until a specific temperature, called the *Curie temperature* $T_C$. The value of $T_C$ is clearly defined for every material. If the temperature of the material is increased above that value the thermal oscillations of the atomic magnets increase significantly and overcome the coupling forces that maintain the alignment of the atomic magnets in the domains. The final effect disturbs the alignment of magnetic moments of adjacent atoms. When a ferromagnetic material is heated above its Curie temperature $T_C$, its magnetic properties are completely changed and it behaves like a paramagnetic material. The permeability of the material drops suddenly to $\mu_r \approx 1$, and both coercivity and remanence become zero (the terms coercivity and remanence will be discussed in the next section). When the material is cooled, the alignment of the atomic magnets in the domains will recover, but the magnetic moments of the domains will be orientated randomly to each other.

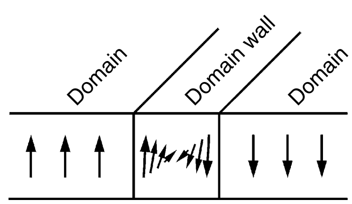


**FIGURE 1.6**
Domain (Bloch) walls.

**TABLE 1.1**

Curie Temperatures of Various Ferromagnetic Elements and Materials

| Material | Curie temperature, $T_C$, [°C] |
|---|---|
| Iron | 770 |
| Cobalt | 1130 |
| Nickel | 358 |
| Gadolinium | 16 |
| Terfenol | 380–430 |
| Alnico | 850 |
| Hard ferrites | 400–700 |
| Soft ferrites | 125–450 |
| Amorphous materials | 350–400 |

Thus, the total external field in the structure will be zero. This means that heating a ferromagnetic material above $T_C$ demagnetizes it completely. The Curie temperatures of various ferromagnetic elements and materials are shown in Table 1.1.

### 1.2.2 Magnetization Processes

Each crystal of a ferromagnetic material contains many domains. The shape, size, and magnetic orientation of these domains depend on the level and direction of the applied external field.

Let us start with an unmagnetized sample of a ferromagnetic material (Fig. 1.7, a). Suppose an external magnetic field $H_{ext}$ in a direction parallel of the domain magnetic moments. With increasing intensity of the applied field the domain walls begin to move (*wall displacement*), first slowly, then quickly, and at the end, in jumps. In the presence of an external field the atomic magnets are subjected to a torque, which tends to align them with the direction of the applied field. The magnetic moments that are in the direction of $H_{ext}$ do not experience a resulting torque. The magnetic moments that are not aligned with $H_{ext}$ are subjected to a torque tending to rotate them in the direction of $H_{ext}$. As a result, the overall domain wall structure becomes mobile and the domains that are in the direction of the applied external field $H_{ext}$ increase in size by the movement of the domain walls into the domains with direction opposite to $H_{ext}$ (Fig. 1.7b). There will be a net magnetic flux in the sample. The *magnetization*, which is the average value per unit volume of all atomic magnets, is increased.

When the applied external field $H_{ext}$ is small, the described domain wall displacements are reversible. When $H_{ext}$ is strong, nonelastic wall displacements occur, which cause hysteresis in the *B-H* relation. Above a certain level of the applied external field, *Barkhausen jumps* of the domain walls occur (Fig. 1.7c). By these jumps, a domain having the direction of the applied field absorbs an adjacent domain with a direction opposite to the applied field.

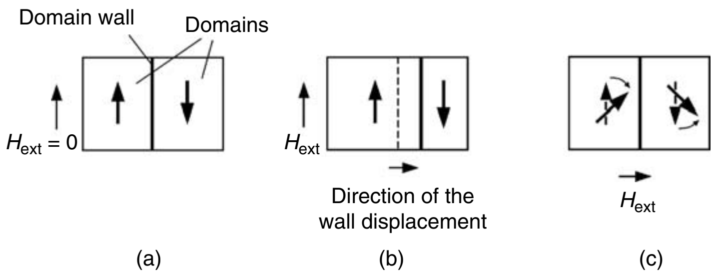


**FIGURE 1.7**
Magnetization of a ferromagnetic sample: (a) without applied external field; (b) with applied external field $H_{ext}$–movement of the domain walls; (c) with applied external field $H_{ext}$–rotation of the domain magnetic moments.

When the strength of the applied external field $H_{ext}$ is increased further, the process of *domain rotation* occurs. The domain magnetic moments rotate in order to align themselves to the direction of $H_{ext}$, thus increasing the magnetization. The process tends to align the domains more to the direction of the applied external field in spite of their initial direction along the crystal axes.

The total magnetization process includes domain wall displacements and jumps and domain rotations. In the case of ferromagnetic metals, at the start the process is realized mainly by means of the wall displacements and jumps, and the rotations of the whole domains take place at the end of the process, doing the final alignment in the preferred directions, defined by the external field.

For further reading, the magnetization processes are described in detail in standard texts [1,2].

### 1.2.3 Hysteresis Loop

Let us suppose a magnetic core with a coil, as shown in Fig. 1.8. At the beginning, the net magnetic flux $B$ in the core, the current $i$ in the coil, and the magnetic field intensity $H$ are zero. Increasing the current in the coil results in applying the field with intensity $H$ according the Ampere's law

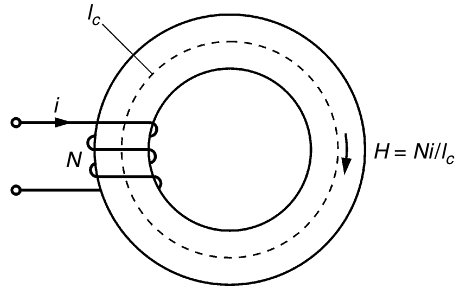


**FIGURE 1.8**
Magnetic core with a coil.

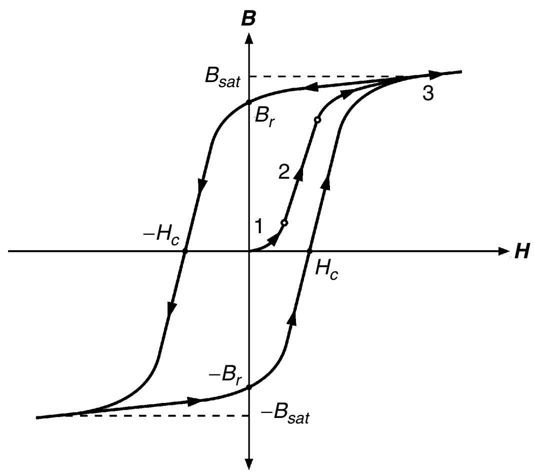


**FIGURE 1.9**
Hysteresis loop and magnetization curve of a ferromagnetic material.

($Hl_c = Ni$, assuming that $H$ is uniform in the core). The first, slowly rising initial section of the magnetization curve, Fig. 1.9, corresponds to reversible domain walls displacements. In the second section of the curve, the induction $B$ increases much more quickly with the increase of $H$ and the curve is steep. The significant increase of $B$ in the second section is explained with the Barkhausen jumps of the domain walls, which occur when the applied external field intensity reaches a necessary level. At the end of this section the structure of the ferromagnetic material contains mainly domains, which are almost aligned along the crystal axes nearest to the direction of the applied external field. The increase of the magnetic flux in the material is not any more possible by domain wall motion. Further increase in $H$ to larger values results in non-significant increase in $B$. and the third section of the magnetization curve is flat. Because the level of $H$ is already much greater than in section 1 and 2, it is enough to initiate the domain rotation process. The contribution of this process to the total magnetic flux is relatively small and gradually decreases. The material reaches saturation and further increase in $H$ results in very small increase in $B$. The maximum value of $B$: the *saturation induction value* $B_{sat}$, is practically reached. All the atomic magnets are aligned along the direction of the applied external field $H$.

Let us observe the process of decreasing $H$, which means decreasing the excitation current $i$ in the coil. The first reaction of reducing $H$ is the rotation of the domains back to their preferred initial directions in parallel with the crystal axes. Further, some domain walls move back in their initial positions, but most of the domain walls remain in the positions reached in the wall displacement process. Thus, the flux $B$ does not return along the same curve, along which it rises with increasing $H$. The new curve, observed with reducing $H$, lags behind the initial magnetization curve. When $H$ reaches zero,

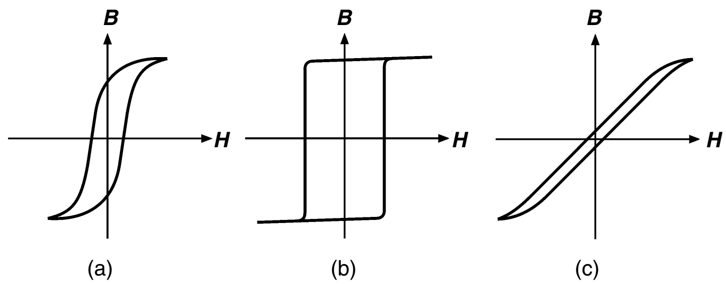


**FIGURE 1.10**
Typical hysteresis loop shapes: (a) round loop, R-type; (b) rectangular loop, Z-type; (c) flat loop, F-type.

residual flux density or remanence, $B_r$, remains mainly due to non-elastic wall displacement process. To reduce this residual flux density $B_r$ to zero, a negative (reversed) field $H$ is necessary to be applied. That field should be sufficient to restore the initial positions of the domain walls. The negative value of $H$ at which $B$ is reduced to zero is called coercive force or coercivity of the material $H_c$. A further increase of $H$ in the opposite direction results in a process of magnetization as the one described above and $B$ reaches saturation level $-B_{sat}$, ($|-B_{sat}| = B_{sat}$). If the current of the excitation coil is repeatedly cycling between the two opposite extreme values, corresponding to the two opposite maximum values of $H$, the hysteresis loop is traced out, as shown in Fig. 1.9.

The hysteresis loop gives the relation between the induction $B$ and the flux intensity $H$ for a closed reversal cycle of magnetization of a ferromagnetic material. The shape of the hysteresis loop is material dependent. Other factors that influence the shape are the excitation frequency and the conditions of the treatment of the material. Some typical hysteresis loops are shown in Fig. 1.10.

The surface of the loop in the $B$–$H$ plane is the energy loss per volume for one cycle.

According to their coercive force $H_c$ the ferromagnetic materials are subdivided in two general classes:

- Soft magnetic materials
- Hard magnetic materials

*Soft magnetic materials* are characterized by an ease of change of magnetic alignment in their structure. This fact results in low coercive force $H_c$ and a narrow hysteresis loop as shown in Fig. 1.11. Soft magnetic materials are of main importance for modern electrical engineering and electronics and are indispensable for many devices and applications. In power electronics most of the magnetic components use cores made from soft magnetic materials.

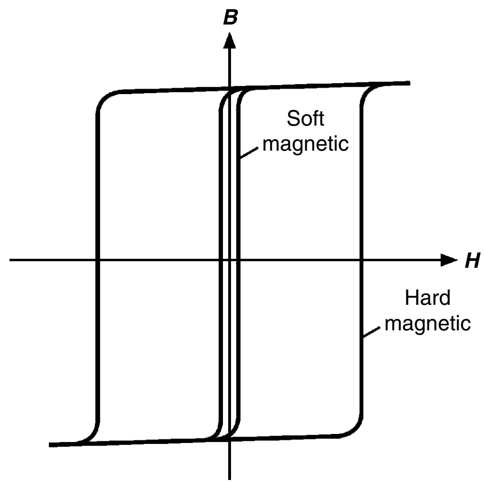


**FIGURE 1.11**
Typical hysteresis loops: (a) a sof magnetic material, narrow loop, low $H_c$; (b) a hard magnetic material, square loop, high $H_c$ and $B_r$.

*Hard magnetic materials* are also called permanent magnets. The initial alignment of the magnetic moments in hard magnetic materials strongly resists any influence of an external magnetic field and the coercive force $H_c$ is much higher than that of soft magnetic materials. Another important property of permanent magnets is their high value of the remanence induction $B_r$. A typical hysteresis loop of a permanent magnet is shown in Fig. 1.11. The permanent magnets produce flux even without any external field. The typical applications of permanent magnets are in electrical motors, generators, sensing devices, and mechanical holding.

The following ranges can be used as approximate criteria for classifying a material as a soft or hard magnetic material [2]:

$H_c < 1000$ A/m soft magnetic material

$H_c > 10\,000$ A/m hard magnetic material

Usually, the values of $H_c$ of most of the used in practice materials are $H_c < 400$ A/m for soft materials and $H_c > 100{,}000$ A/m for hard magnetic materials.

### 1.2.4 Permeability

Permeability is an important property of magnetic materials and therefore we will discuss it in detail. The relative permeability $\mu_r$ introduced in Section 1.1 has several different interpretations depending on the specific conditions of defining and measuring it. The index $r$ is omitted and only the corresponding index is used in denoting the different versions: amplitude permeability $\mu_a$, initial permeability $\mu_i$, effective permeability $\mu_e$, incremental permeability $\mu_{in}$, reversible permeability $\mu_{rev}$, and complex permeability $\underline{\mu}$.

*Amplitude permeability* $\mu_a$ is the relative permeability under alternating external field $H$, which gives the relation between the peak value of the induction $B$ and the magnetic field $H$. Its general definition is

$$\mu_a = \frac{1}{\mu_o}\frac{\hat{B}}{\hat{H}} \tag{1.11}$$

where
$\hat{B}$ is the amplitude induction value averaged out over the core cross-section
$\hat{H}$ is the amplitude field parallel to the surface of the core

The *initial permeability* $\mu_i$ is the relative permeability of the magnetic material when the applied magnetic field $H$ is very low:

$$\mu_i = \frac{1}{\mu_0}\frac{\Delta B}{\Delta H}(\Delta H \to 0) \tag{1.12}$$

For practical purposes the value obtained at a small field $H$ is standardized [2], e.g., as the permeability at $H$ = 0.4 A/m (see Fig. 1.12).

If there is an air gap in a closed magnetic circuit, the apparent total permeability of the circuit is called *effective permeability* $\mu_e$, which is much lower than the permeability of the same core without an air gap. The effective permeability depends on the initial permeability $\mu_i$ of the magnetic material and the dimensions of the core and the air gap. For cores with relatively small (short) air gaps the effective permeability is given by

$$\mu_e = \frac{\mu_i}{1 + \frac{A_g \mu_i}{l_c}} \tag{1.13}$$

where
$A_g$ is the cross-sectional area of the air gap
$l_c$ is the effective length of the magnetic path

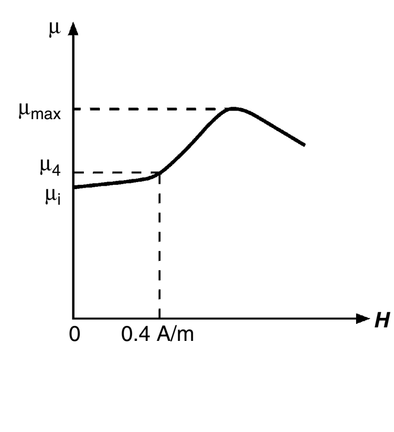


**FIGURE 1.12**
Definition of $\mu_i$, $\mu_4$, and $\mu_\Delta$ dependent on the field $H$.

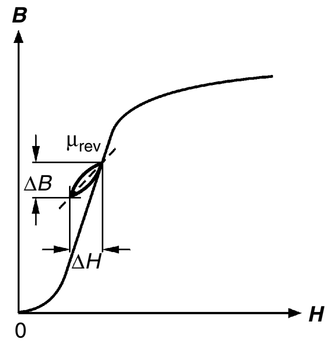


**FIGURE 1.13**
Definition of the reversible permeability $\mu_{\text{rev}}$.

If the air gap is long, some part of the flux passes outside the air gap and this additional flux results in an increased value of the effective permeability in comparison with Equation (1.13). Therefore Equation (1.13) is valid only when fringing permeability is neglected. The effective permeability is also known as the permeability of an equivalent homogeneous toroidal core.

*Incremental permeability* $\mu_\Delta$ is defined when an alternating magnetic field $H_{\text{AC}}$ is superimposed on a static magnetic field $H_{\text{DC}}$. The hysteresis loop follows a minor loop path. The incremental permeability is

$$\mu_\Delta = \left( \frac{1}{\mu_0} \frac{\Delta B}{\Delta H} \right)_{H_{\text{DC}}} \tag{1.14}$$

The limiting value of the incremental permeability $\mu_{\text{in}}$, when the amplitude of the alternating field excitation $H_{\text{AC}}$ is very small, is termed *reversible permeability* $\mu_{\text{rev}}$ (see Fig. 1.13):

$$\mu_{\text{rev}} = \frac{1}{\mu_0} \frac{\Delta B}{\Delta H}, \quad \Delta H \to 0 \tag{1.15}$$

### 1.2.4.1 Complex Permeability

In practice, we never have an ideal inductance when the core is made from a magnetic material. Under sinusoidal excitation there is a phase shift between the fundamental components of the induction $B$ and the magnetic field $H$. By using a complex quantity for the relative permeability, consisting of a real part and an imaginary part, these effects are easily presented. The imaginary part of the *complex permeability* $\underline{\mu}$ is associated with the losses in the material. There are two different forms of the complex permeability $\underline{\mu}$.

- Series representation, according to the series equivalent circuit of magnetic component shown in Fig. 1.14a:

$$\underline{\mu} = \mu_s' - \mathrm{j}\,\mu_s'' \tag{1.16}$$

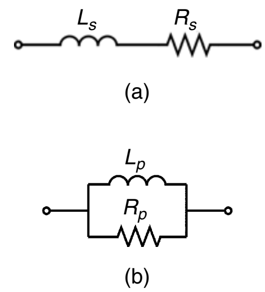


**FIGURE 1.14**
Series and parallel equivalent circuits.

where
$\mu_s'$ and $\mu_s''$ are the real and imaginary parts of the complex permeability

- Parallel representation, according to the parallel equivalent circuit shown in Fig. 1.14b):

$$\frac{1}{\underline{\mu}} = \frac{1}{\mu_p'} + j\frac{1}{\mu_p''} \tag{1.17}$$

where
$\mu_p'$ and $\mu_p''$ are the real and imaginary parts of the complex permeability

In Fig. 1.15 the complex permeability is represented by the series terms in the frequency domain. These values are often given in the data to describe the behavior of the material at very low induction levels (signal applications). The graphs of the real and imaginary parts versus frequency are often shown to describe the frequency behavior of the material. The values of the real and imaginary parts of the complex permeability in the series presentation for a given frequency can be calculated form the measured inductance $L_s$ and resistance $R_s$ of the coil of it series equivalent circuit.

The parallel representation has the advantage that the loss associated part $\mu_p''$ does not change when an air gap is added in the magnetic circuit. Usually in applications the induction $B$ is known, which allows the calculation of the losses directly by using $\mu_p''$. The parallel representation is more often used in power applications.

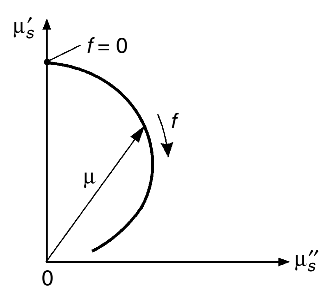


**FIGURE 1.15**
Complex permeability presented by the series terms in the frequency domain.

Depending on the application and purpose, the series or parallel presentation may be used. The following expressions give the relation between the series and parallel presentation parts of the complex permeability:

$$\mu_p' = \mu_s'(1 + \tan^2 \delta) \tag{1.18}$$

$$\mu_p'' = \mu_s''\left(1 + \frac{1}{\tan^2 \delta}\right) \tag{1.19}$$

In Equations (1.18) (1.19) $\delta$ is the *loss angle*, which is also the phase lag of the induction $B$ with respect to the applied magnetic field $H$. The tangent of the loss angle $\delta$ is given by the expression

$$\tan \delta = \frac{\mu_s''}{\mu_s'} = \frac{\mu_p'}{\mu_p''} \tag{1.20}$$

The quantity tan $\delta$ is also the ratio of the equivalent series resistance of a coil (neglecting copper resistance) to its reactance, which is the reciprocal value of quality factor of the inductance:

$$\tan \delta = \frac{R}{\omega L} = \frac{1}{Q} \tag{1.21}$$

The complex permeability is mainly used in signal electronics and for low induction levels and is less often used in power electronics. In power electronics the magnetic materials have a nonlinear frequency behavior. We would like to warn the reader that if the ferrite losses at high induction levels are estimated by $\mu'$ and $\mu''$ values, which are relevant at low induction levels, then the losses can be severely underestimated. The reason is that the losses in the ferrites increase more than the square of the induction $B$.

#### *1.2.4.2 Hysteresis Material Constant*

The losses of some ferrite grades are described using the hysteresis constant $\eta_B$, which is defined at low induction levels. The hysteresis constant $\eta_B$ is defined by the following expression [7]:

$$\Delta \tan \delta_h = \mu_e \eta_B \Delta \hat{B} \tag{1.22}$$

where

$\Delta \hat{B}$ is the amplitude of the induction $B$

$\mu_e$ is the effective permeability

The hysteresis losses increase when the induction in a core increases. The contribution of the hysteresis losses to the total losses can be estimated by means of the results of two measurements, usually at the induction levels 1.5 mT and 3 mT [4]. By these measurements the hysteresis constant $\eta_B$ is found from

$$\eta_B = \frac{\Delta \tan\delta}{\mu_e \Delta \hat{B}} \tag{1.23}$$

and then it is used to find $\delta_h$ by Equation (1.22).

The consequence of this behavior is that at low $B$ values the losses tent to increase with $B^2$, whereas at large $B$ values the dependence is close to $B^3$.

## 1.3 Magnetic Circuits

### 1.3.1 Basic Laws for Magnetic Circuits

According to Ampere's law, the sum of the MMF around a closed magnetic loop is zero:

$$\sum MMF_{\text{loop}} = 0, \quad \sum MMF_{\text{source}} = \sum MMF_{\text{drop}} \tag{1.24}$$

This requirement is analogous to the Kirchoff's voltage law. The $\text{MMF}_{\text{drop}}$ for an element of a magnetic circuit is

$$MMF_{\text{drop}} = Hl \text{ [A} \cdot \text{turns]} \tag{1.25}$$

Substituting $H = B/\mu$ and $B = \Phi/A_c$ results in the following expressions:

$$\int MMF_{\text{drop}} = \Phi \frac{l}{\mu A_c} = \Phi \Re = \frac{\Phi}{\Lambda} \tag{1.26}$$

$$Hl = \Phi \Re \quad \Rightarrow \quad F = \Phi \Re \tag{1.27}$$

In Equation (1.26) the magnetic flux $\Phi$ is analogous to current $I$, and the quantity $\Re = l/\mu A_c$ is analogous to resistance $R$. The quantity $\Re = l/\mu A_c$ [A · turns/Wb] is called *reluctance* and we will use the symbol $\Re$ for it. The quantity $1/\Re$ [Wb/A · turns] is called *permeance* $\Lambda$ of the magnetic path (in soft ferrites data this value is often denoted as $A_L$ value).

For a magnetic circuit with an air gap (Fig. 1.16), by splitting the left side into two terms and assuming that $H$ is almost uniform in both mediums, the Ampere's law can be written as

$$H_c l_c + H_g l_g = NI \tag{1.28}$$

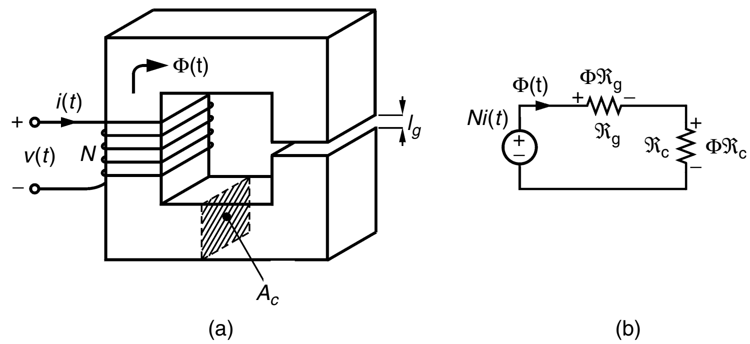


**FIGURE 1.16**
Magnetic circuit with an air gap: (a) physical geometry; (b) equivalent circuit scheme.

where
$H_c$ and $H_g$ are the field intensity in the core and in the air gap, respectively
$l_c$ is the magnetic path length in core
$l_g$ is the length of air gap

Considering Fig. 1.16, the application of Gauss's law for a closed surface crossing the core and the air gap and including the total transition surface between them, gives the expression

$$\int \boldsymbol{B}_c \cdot \mathrm{d}\boldsymbol{S} + \int \boldsymbol{B}_g \cdot \mathrm{d}\boldsymbol{S} = 0 \tag{1.29}$$

which yields

$$\Phi_c = \Phi_g = \Phi \tag{1.30}$$

Equation (1.28) can be rewritten as

$$\Phi_c \Re_c + \Phi_g \Re_g = \Phi(\Re_c + \Re_g) = NI \tag{1.31}$$

where
$\Phi_c$ is the magnetic flux in the core
$\Phi_g$ is the magnetic flux in the air
$\Re_c$ is the reluctance of core path
$\Re_g$ is the reluctance of the air gap

Equations (1.29) and (1.30) are valid only for small air gaps. At larger air gaps, the flux tends to the outside. In contrast to electrical circuits true "insulation" is not present, as the relative permeability of air equals 1, which is nonzero.

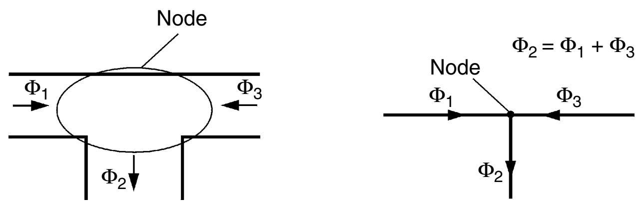


**FIGURE 1.17**
Application of Gauss's law to a node of a magnetic circuit.

The application of Gauss's law for a node of a magnetic circuit gives the result that the algebraic sum of fluxes coming out of the node is equal to zero, as it is shown in Fig. 1.17:

$$\oint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 0 \quad \Rightarrow \quad \sum_{i=1}^{n} \Phi_i = 0 \tag{1.32}$$

Equation (1.32) is analogous to Kirchoff's current law.

For further reading, magnetic circuits and components are presented in a suitable way for the needs and the applications of power electronics in textbooks on power electronics [3,4,5]. Electromagnetic concepts and applications are described in detail in Marshall et al. [6].

## 1.3.2 Inductance

### *1.3.2.1 Flux Linkage*

First, we will define the term *flux linkage*, $\Psi$ (flux linked to all turns). The instantaneous voltage across a coil can be presented as

$$v(t) = R\,i(t) + \frac{\mathrm{d}\Psi(t)}{\mathrm{d}t} = R\,i(t) + e(t) \tag{1.33}$$

where $R$ is the ohmic resistance of the coil, $i(t)$ is the coil current and $e(t)$ is the electromotive force.

From that expression we define the term $\Psi(t)$:

$$\Psi(t) = \int e(t)\,dt = \int (v(t) - R\,i(t))\,\mathrm{d}t \tag{1.34}$$

with dimension [Weber] or [V · s].

We prefer [V · s], as it reminds that the quantity is a flux linkage and not a physical flux.

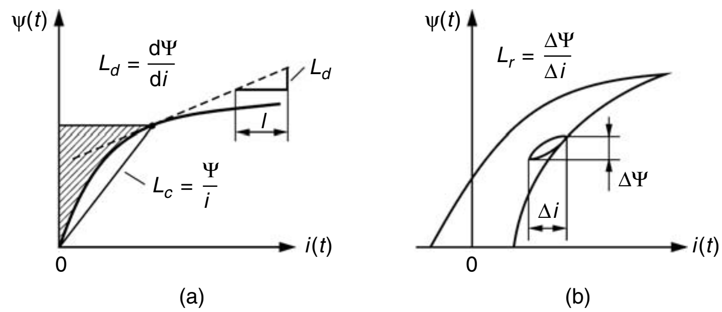


**FIGURE 1.18**
Flux linkage Ψ as a function of current $i$ and definitions of $L_c$, $L_d$, and $L_r$.

### *1.3.2.2 Inductance: Definitions*

The term *inductance* can be defined in different ways with respect to the nonlinearity of the *B-H* dependence. For simplicity, we do *not consider core losses* in this section. Here we explain the different definitions and presentations of the term *inductance*.

*Chord Inductance or Amplitude Inductance*

The slope of the chord in the curve $\Psi = \Psi(t)$ is called *chord* inductance or *amplitude* inductance (see Fig. 1.18a), and is denoted $L_c$, $L_a$, or simply $L$:

$$L_c = \frac{\Psi}{i} \text{ [H] (Henry) or } [\Omega \cdot \text{s}] \tag{1.35}$$

*Differential Inductance*

The (derivative) of the flux linkage $\Psi = \Psi(i)$ is the *differential* inductance $L_d$. This inductance is observed when small signals are superimposed to the coil current $i$.

$$L_d = \frac{d\Psi}{di} \tag{1.36}$$

Note that with material having hysteresis losses, see Fig. 1.18b, a minor loop is observed resulting in a lower small signal inductance, called *reversible inductance*:

$$L_r = \frac{\Delta\Psi}{\Delta i} \tag{1.37}$$